Resonant Frequency and Q Factor of Axisymmetric Composite Microwave Cavities

Ahmed A. Kishk, *Fellow, IEEE*, Darko Kajfez, *Life Senior Member, IEEE*, and Siva Chebolu, *Member, IEEE*

***Abstract*—Resonant frequency and unloaded Q factor of composite microwave cavities are computed using surface integral equations for axisymmetric objects. The equivalence principle is used to formulate the problem so that the unbounded space Green's function can be utilized. The numerical results are verified experimentally for many samples of conducting cavities and dielectric resonators inside conducting cavities. Also, cases with a sector of the rotational cavity are considered by introducing a conducting corner. The method allows the computation of the stored energy in each dielectric region and the unloaded Q factor.**

***Index Terms*—Cavity resonators, dielectric resonators, integral equations, Q factors, resonant frequencies.**

I. Introduction

CERAMIC materials of high dielectric constant and low loss can exhibit a very high-Q factor. Therefore, they have been widely used as resonators in microwave filters and oscillators. To understand the behavior of these elements and to determine the best coupling mechanism, it is necessary to determine the electromagnetic-field distribution inside the cavity. The resonant frequency and Q factor have to be computed accurately before the electromagnetic fields in the vicinity of the resonators can be obtained. It is, therefore, important to be able to determine the resonant frequencies and Q factors of the desired modes.

Many different approaches to the analysis of dielectric resonators (DRs) have been described in the literature [1]–[22]. Some of these methods are based on simplifications of the geometry, such as the perfect magnetic conducting (PMC) walls method [1]–[3]. Dielectric waveguide methods [4], as well as their perturbation corrections and the variation improvements [5], for cylindrical resonators have also been developed. In 1975, Van Bladel reported a rigorous asymptotic method for evaluating the modes of DRs of arbitrary shape and high permittivity [6], [7]. In addition, radial and axial mode-matching methods [9] for shielded resonators, as well as asymptotic expansion methods [10] have been reported. Also, general mode-matching approaches using Green's dyadic functions or transverse modes in expanding the interior and exterior fields [11] have been successful approaches for this kind of problem. Glisson *et al.* [12], Kajfez *et al.* [13], and Kishk *et al.* [14] introduced the use of the method of moments (MoM) for the analysis of an open DR. The method was applied to dielectric bodies of revolution of arbitrary cross section and is useful for any azimuthal variation. Their results indicate that their method has yielded highly accurate values of resonant frequencies and Q factors for homogeneous DRs. Also, the finite integration method was used for axisymmetric shielded resonators [15]. Another recently reported approach to the electromagnetic resonance of open DRs is the null-field method [16], [17] developed by Zheng and Strom. In that approach, the resonance problem is solved by searching for zeros of the determinant of the so-called Q-matrix for the DRs.

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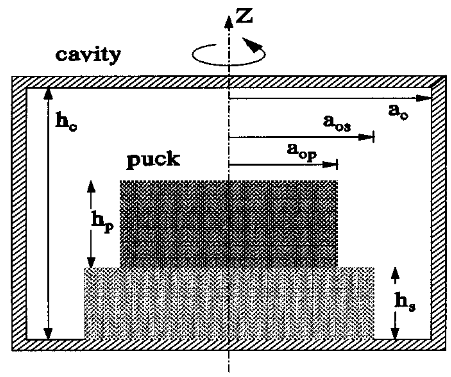


Fig. 1. Example of the original cavity.

A previously developed numerical method [18] based on the MoM is used here to search for the complex resonant frequency, from which the resonant frequency and radiation Q factor can be computed, of both homogeneous and inhomogeneous DRs in free space. The geometries considered here are rotationally symmetric so the body of revolution approach is employed. The equivalence principle is used and the surface integral equations are formulated for the problem. The MoM is then used to reduce the integral equations to a matrix equation. The natural resonant frequencies are defined as the frequencies at which the determinant of the moment matrix vanishes. Also, since the rotationally symmetric structure supports independent azimuthal modes, the azimuthal variation of the unknown equivalent currents is expanded in Fourier series. This allows one to search for the zeros of the moment matrix for modes having a particular azimuthal variation.

To compute the Q factor of nonradiating cavities, we consider a lossless structure and search for the resonant frequencies. Once the resonant frequency is known, the surface currents are computed, which are then used to compute field distributions inside the cavity. The perturbation method is then employed to compute the stored energy and dissipated power from which we compute the Q factor.

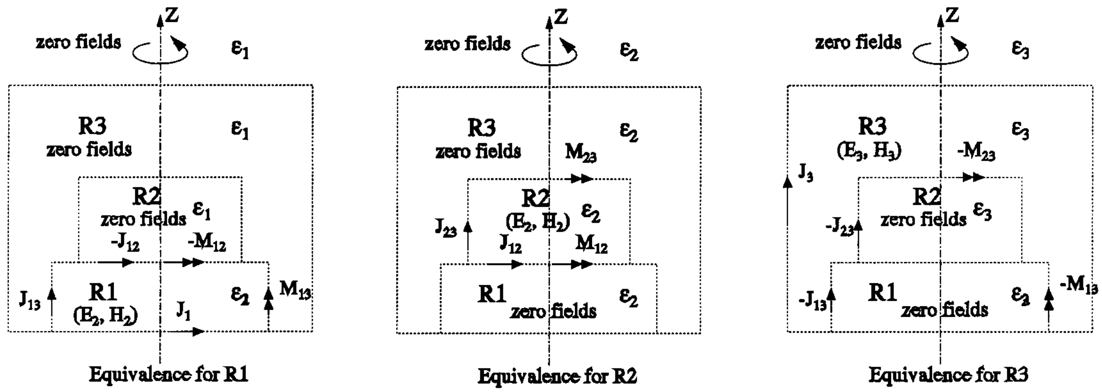


Fig. 2. Equivalent problems.

II. Equivalence Principle

The problem is formulated based on the surface equivalence principle [18]. To illustrate the procedure, an example of a DR in a cavity is considered, as shown in Fig. 1, as the original problem, which consists of two dielectric disks (the puck and support) inside a conducting cavity. This problem consists of three regions, namely: 1) the puck region; 2) the support region; and 3) the free-space cavity volume. To formulate the problem using the surface equivalence principle, three equivalent problems are needed. These equivalent problems are presented graphically, as shown in Fig. 2. In summary, each region has an unbounded equivalent problem filled with materials of the same type as the original material of the region with nonzero fields and zero field outside of the particular region. The surface surrounding the region carries equivalent electric and magnetic currents to compensate for the discontinuity in the assumed field distribution. The equivalent currents on the dielectric surfaces of each equivalent problem must satisfy the continuity of the tangential fields. Notice that we only consider the interior conducting surface of the cavity.

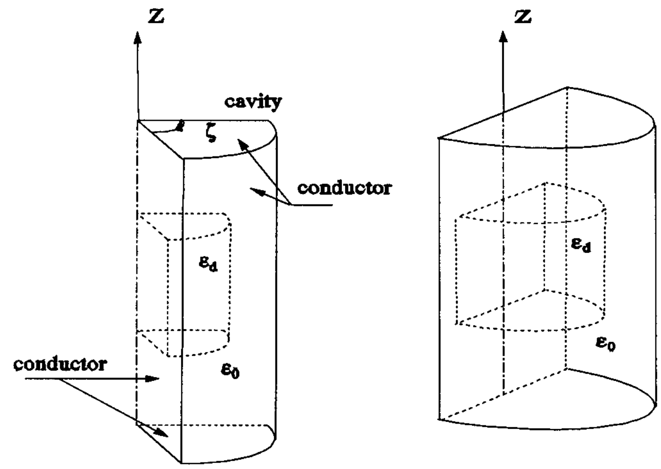
Surface integral equations are obtained from the boundary conditions on all boundaries in terms of unknown surface currents. The equivalent surface currents for body of revolution can be expressed as

$$\boldsymbol{J}_s(r') = \sum_{m,\,i} \left[I^t_{mi}\hat{u}_t + I^\phi_{mi}\hat{u}_\phi \right] f(t_i) e^{jm\phi} \tag{1}$$

$$\boldsymbol{M}_s(r') = \eta_0 \sum_{m,\,i} \left[V^t_{mi}\hat{u}_t + V^\phi_{mi}\hat{u}_\phi \right] f(t_i) e^{jm\phi} \tag{2}$$

where $\boldsymbol{J}$ and $\boldsymbol{M}$ are the equivalent electric and magnetic currents, respectively. Surface coordinates t and ϕ are introduced on the surface, where t is the arc length along the generating curve and ϕ is the azimuthal angle measured from the x–z-plane. Also, $\hat{\boldsymbol{u}}_t$ and $\hat{\boldsymbol{u}}_\phi$ are the unit vectors tangential to the body surface in the direction t and ϕ. $f(t)$ is the triangle basis function and m is the index of the azimuthal mode. More details can be found in [18]. For each azimuthal mode m, the integral equations are reduced to a system of matrix equations in the form

$$\left[\overline{T}_m\right]\left[\overline{C}_m\right] = \left[\overline{V}_m\right], \qquad m = 0, \pm1, \pm2, \ldots \tag{3}$$



(a) (b)

Fig. 3. Geometries of the sectorial cavity. (a) Circular sectorial cavity with angle ζ. (b) Circular sectorial cavity with $\zeta = 180°$.

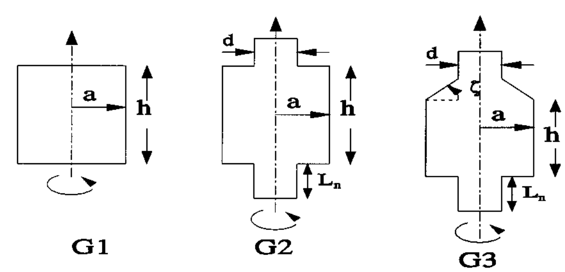


Fig. 4. Cross sections of different configurations of conducting cavities.

where $\overline{T}_m$ is a square matrix, $\overline{C}_m$ is a column vector for the unknown coefficients, and $\overline{V}_m$ is the excitation vector. The solution of (3) provides the unknown current coefficients on all the object boundaries. In general, the current coefficients can be used to compute the fields as

$$\boldsymbol{E}(\boldsymbol{r}) = \frac{-j}{4\pi\omega\epsilon} \int_s \left[k^2 \boldsymbol{J}_s + (\nabla_s \cdot \boldsymbol{J}_s)\nabla - j\omega\epsilon \boldsymbol{M}_s \times \nabla \right] \times G(|\boldsymbol{r} - \boldsymbol{r}'|)\, ds' \tag{4}$$

where G is the free-space Green's function. The duality can be used to obtain the magnetic-field expressions. The cylindrical near electric-field component expressions due to the surface current components are given in the Appendix.

TABLE I
TE_{111}-MODE RESONANT FREQUENCY AND Q FACTOR FOR A CAVITY FILLED WITH OIL

Case	σ (S/m)	ϵ_r	tan δ	computed, f_o (GHz)	measured or exact, f_o (GHz)	computed Q_o	measured or exact, Q_o
G1(exact)	1.57E7	1.0	0.0	15.054	15.051	4556	4572
G2	6.17E7	1.0	0.0	14.957	14.820	8462	9803±123
G3	1.57E7	1.0	0.0	14.730	14.681	4408	N/A
G1(exact)	1.57E7	2.2	0.001	10.156	10.150	782	789
G2	6.17E7	2.2	0.001	10.084	10.145	154	155.5
G3	1.57E7	2.2	0.001	9.935	9.971	783	790±17

TABLE II
RESONANT FREQUENCY AND Q FACTOR OF ISOLATED DR

mode	f (computed)GHz	f (measured) GHz	Q(computed)	Q(measured)
$TE_{01\delta}$	4.854	4.85 [21] , 4.85 [5]	40.9	46.4 [21], 51 [5]
$HEM_{11\delta}$	6.334	6.33 [21], N/A [5]	31.9	30.3 [21], N/A [5]
$HEM_{12\delta}$	6.654	6.612 [21], 6.64 [5]	51.5	43.3 [21], 64 [5]
$TM_{01\delta}$	7.536	7.494 [21], 7.60[5]	77.2	58.1 [21], 86 [5]

To search for the resonant natural frequencies (frequencies for which the system can have a response with no excitation), replace $j\omega$ by the complex frequency s, where $j = \sqrt{(-1)}$ and ω is the radial frequency [12]. These frequencies can be determined by searching for the frequencies at which the determinant of the matrix $[\overline{T}_m]$ is zero. For a lossless system (no losses are considered for the materials), the roots of

$$\det\left[\overline{T}_m\right] = 0 \tag{5}$$

could be real or complex. If the structure is not enclosed by a conducting cavity, the system will have some radiation losses and the resonant frequency is complex. The resonant frequency is expressed as

$$s_{m,\,\upsilon} = -\sigma_{m,\,\upsilon} + j\omega_{m,\,\upsilon}. \tag{6}$$

The radiation Q factor for resonators in the free space is computed as

$$Q_{m,\,\upsilon} = -\frac{\omega_{m,\,\upsilon}}{2\sigma_{m,\,\upsilon}}. \tag{7}$$

For closed cavities, lossless materials are considered and because the cavity is not radiating, the resonant frequency is real and $\sigma_{m,\,\upsilon} = 0$. To compute the total Q factor at the resonant frequency, the field distribution is computed within the cavity and the perturbation method is used to compute the stored energy and the dielectric and conduction power losses in terms of the known loss tangent ($\tan\delta$) and surface conductivity of the constituent materials. The dielectric Q factor (Q_d) and conducting Q factor (Q_c) are related to the total unloaded Q factor (Q_u) as follows:

$$\frac{1}{Q_u} = \frac{1}{Q_c} + \frac{1}{Q_d}. \tag{8}$$

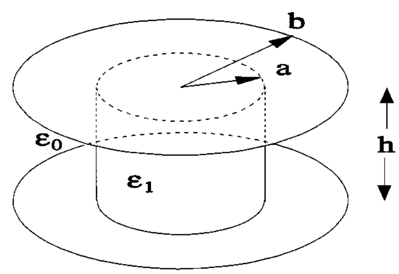


Fig. 5. Cylindrical DR between parallel plates.

Using the numerical method, one can obtain the percentage of electromagnetic stored energy in each region. In some applications, certain modes can be excited without a change in the resonant frequency by partitioning the cavity by a conducting corner with the apex passing by the axis of symmetry, as shown in Fig. 3. The image theory is then used and the stored energy is computed for the cavity part, which is proportional with the volume of the partitioned part to the full symmetric cavity. The dielectric power loss is proportional to the volume, but the conduction losses will consist of two parts. The first part is proportional with the volume ratio and the other part is due to the new conducting materials added to form the corner. This means that the total stored energy is reduced and the power losses are increased, which will result in a reduction of the Q factor. The dielectric Q factor of each dielectric region can also be computed.

III. RESULTS

A. Conducting Cavity Filled With Oil

Circular cylindrical resonant cavities are used for precision measurement of complex permittivity of liquids [20]. The cavity dimensions ($a = 12.7$ mm, $h = 25.4$ mm) are chosen to res-

TABLE III
RESONANT FREQUENCY AND Q FACTOR OF THE CYLINDRICAL DR IN FIG. 5

TE_{011}	f_o GHz	Q-factor
Computed (open)	2.934	3972.5
Computed (closed)	2.937	4113
Computed (closed) [19]	2.937	4415
Measured open [19]	2.937	4200

onate around 10 GHz with the liquid of permittivity 2.2 in presence. The liquid is pressured through a narrow pipe to fill the cavity and flow out from a narrow pipe from the other side of the cavity, as shown in Fig. 4. The pipe diameter $d = 12.7$ mm and length $L_n = 10$ mm. To reduce the vertex of the liquid in the cavity at the pipe entrance, a tapering with $\zeta = 45°$ is introduced. This tapering increased the cavity volume and caused a reduction of the resonant frequency. The numerical model of the narrow pipe is checked with a short-circuit and an open-circuit termination at each end. As the cavity resonant frequency is much lower than the cutoff frequency of the narrow pipes, the resonant frequency and Q factor are not sensitive to the opening or closing of these pipes. The resonant frequency and Q factor, computed and compared with some measured results, are shown in Table I. The resonant frequency and Q factor of the cylindrical cavity are computed exactly using analytical expressions, as shown in Table I. Excellent agreement is obtained. For an air-filled cavity, the comparison between the measured Q factor and computed Q factor has a larger error because of the uncertainty in the value of the conductivity. When the cavity is filled with liquid, the dielectric Q factor (Q_d) is much lower than that of the conductor (Q_c). Therefore, Q_d dominates the value of the unloaded (Q_u). The computed and measured Q_u are in much better agreement with each other.

B. Isolated DR

For a dielectric cylinder disc in the free space with a radius $a = 5.25$ mm, height $= 4.6$ mm, and dielectric constant $\epsilon_r = 38$, the resonant frequency and radiation Q factor are given in Table II for different modes. The computed values are in excellent agreement with the computed values using other methods in [12] and [21]. Very good agreement between computed and measured frequencies is also obtained. However, the Q factor shows a large difference between computed and measured values. One can even see differences between the values obtained experimentally in [5] and [21]. One can conclude that the measurements of the small Q factor are less reliable than those with high-Q factors, particularly when the 3-dB method is used.

C. Cylindrical DR Between Parallel Plates

This constitutes a partially open resonating structure, also known as a Courtney holder. A cylindrical DR is located between two silver-coated parallel conducting plates, as shown in Fig. 5, with $\varepsilon_{r1} = 37.5$, $\tan\delta = 0.0001$, the radius of the DR is $a = 12.13$ mm, $b = 10a$, and the height of the DR is $h = 11.15$ mm. The present method is used to compute the resonant frequency and Q factor for the nonradiating TE_{011} mode using two parallel plates and a closed cavity, as in [19], with radius $b = 10a$. The results are shown in Table III. The difference between resonant frequencies (0.85%) based on infinite plates and finite plates of radius $10a$ is not clear.

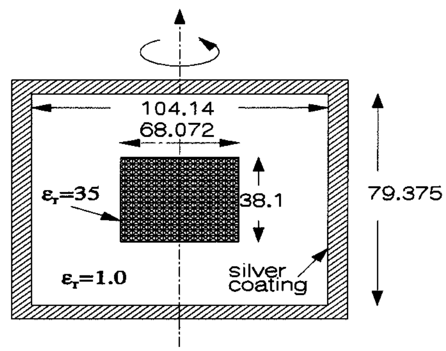


Fig. 6. Geometry of a DR disc in a conducting cavity (all dimensions in millimeters).

D. Simple Shielded DR

The code has been verified for the cylindrical conducting cavities filled with homogeneous dielectric material, as shown above. The code is also suited for computation of the resonant frequency and the Q factor of a DR fully enclosed by a conducting cavity. The geometry of such a structure is shown in Fig. 6. The dielectric disk with $\varepsilon_{r1} = 35$, $\tan\delta = 0.00002$, $a = 34.036$ mm, and $h = 38.1$ mm, is centered in a conducting cavity coated with silver paint with a radius of 52.07 mm and a height of 79.375 mm. The computed resonant frequency and Q factor for different modes are as shown in Table IV. Also shown are the filling factors, namely, the ratios between the stored energy in a certain region and the total stored electric energy in the cavity [5, p. 332].

E. Circular Sectoral Cavities

The present numerical procedure for analyzing the bodies of revolution can also be applied to the cavities that occupy only a sector of the full circle, such as shown in Fig. 3. If the cavity is split through its axis of symmetry in two equal halves by a conducting plate (with a corner angle of 180°), the $TE_{01\delta}$, $HEM_{11\delta}$, and $HEM_{12\delta}$ modes will have the same resonant frequency without any disturbance of the field distribution or the resonant frequency. However, the Q factor will be reduced due to the added conducting losses from the plate. The Q factor for the $TE_{01\delta}$ mode in Table IV is reduced to 18 957, for the $HEM_{11\delta}$ mode, it is reduced to 15 897, and for the $HEM_{12\delta}$

TABLE IV
RESONANT FREQUENCY AND Q FACTOR OF THE SHIELDED RESONATOR IN FIG. 6

Mode	f_0 (MHZ)	Q-factor	DR filling factor	Cavity filling factor
$TE_{01\delta}$	816.0458	30354	0.992	0.008
$HEM_{12\delta}$	941.216	34148	0.979	0.021
$HEM_{11\delta}$	1008.668	43295	0.83	0.17
$TM_{01\delta}$	1066.837	42314	0.84	0.16

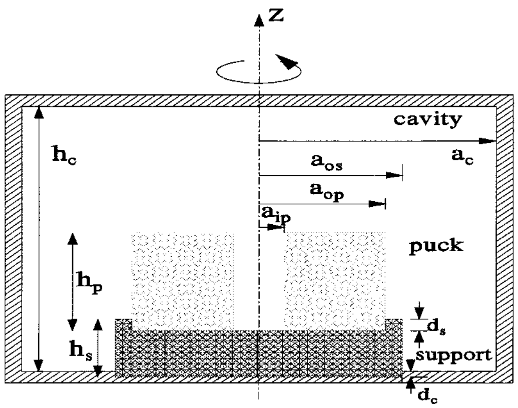


Fig. 7. Composite DR inside a cavity.

TABLE V
PUCK AND SUPPORT DIMENSIONS IN THE CAVITY FROM FIG. 7

sample	a_{ip} (mm)	a_{op} (mm)	h_p (mm)	h_s (mm)
1	4.85	32.910	30.480	31.699
2	4.85	32.855	29.845	31.699
3	4.864	32.880	29.210	31.699
4	4.85	32.910	30.480	29.591
5	4.85	32.855	29.845	29.591
6	4.864	32.880	29.210	29.591

mode, it is reduced to 1446. If the conducting plate becomes a corner with an angle less than 180°, only the $\mathrm{TE}_{01\delta}$ mode can be supported and the other three modes in Table IV will be suppressed. For a 90° corner of conducting materials (one-quarter of the full cavity), the Q factor of the $\mathrm{TE}_{01\delta}$ mode is reduced further to 12 490.

F. Composite Microwave Cavity

A cavity with multiple dielectric materials is shown in Fig. 7. Several samples are considered with the dimensions and parameters given in Table V. The cavity is silver plated with $a_c = 50.8$ mm, $h_c = 76.2$ mm, and the support bored of depth $d_c = 0.4572$ mm. The cavity encloses a tubular DR (puck) and a supporting disk. The puck has a dielectric constant $\varepsilon_{rp} \approx 36$ at 900 MHz, $\tan\delta_p = 0.0002$ at 900 MHz, $a_{ip} = 4.8514$ mm, $a_{op} = 32.9057$ mm, and $h_p = 30.48$ mm. The support has $\varepsilon_{rs} = 3.15$ at 1 MHz, $\tan\delta_s = 0.0013$ at 1 MHz, $a_{os} = 38.1$ mm, $h_s = 31.7$ mm, and groove in the support of depth $d_s = 6.27$ mm. The computed and measured resonant frequency and Q factors for the $\mathrm{TE}_{01\delta}$ mode are given in Table VI.

TABLE VI
COMPUTED AND MEASURED RESONANT FREQUENCY AND UNLOADED Q FACTOR OF THE CAVITY IN FIG. 7

Sample	Measure Q_u	Measured frequency (MHz)	Computed Q_u	Computed frequency (MHz)
1	22572± 098	866.289	22147.52	860.8744
2	22114± 094	861.578	22051.56	865.5408
3	22028± 152	865.578	21914.10	869.0690
4	22692± 145	859.108	22540.23	860.5463
5	22508± 145	862.093	22428.90	865.2626
6	22285± 146	866.289	22274.46	868.9650

The measurement of the Q factor is performed with a network analyzer and the data were processed by the program Qzero [22].

IV. CONCLUSIONS

Axisymmetric composite cavities are analyzed using the surface integral equations and the MoM to compute the resonant frequency and unloaded Q factor. Very good agreement between the computed and measured results is obtained. The resonant frequency is predicted within 0.5% error and the Q factor is predicted with less than 2% for the closed cavities, unless the cavity losses are not accurate. The Q factor of open resonators shows larger errors compared to measured data because the Q factor is low and the available measured data are not accurate enough. The present method is flexible in analyzing a wide range of structures. We have also shown that it is easy to implement this method in MoM codes that were developed for scattering problems.

APPENDIX
NEAR-FIELD EQUATIONS IN THE CYLINDRICAL COORDINATE SYSTEM

The cylindrical near electric-field components are given as

$$E_\rho\left(J_{mi}^t\right) = \frac{-j\eta J_{mi}^t}{4\pi}\Bigg[+k\int_{t_i} dt\big(\rho_i f_i(t)\big)\sin\nu_i G2 - \frac{1}{k}\int_{t_i} dt\frac{\partial}{\partial t}\big(\rho_i f_i(t)\big)(\rho G1' - \rho_i G2')\Bigg] \quad \text{(A1)}$$

$$E_\phi\left(J_{mi}^t\right) = \frac{-j\eta J_{mi}^t}{4\pi}\left[jk\int_{t_i} dt\big(\rho_i f_i(t)\big)\sin\nu_i G3 + \frac{j}{k}\int_{t_i} dt\frac{\partial}{\partial t}\left(\rho_i f_i(t)\right)\rho_i G3'\right] \tag{A2}$$

$$E_z\left(J_{mi}^t\right) = \frac{-j\eta J_{mi}^t}{4\pi}\left[+k\int_{t_i} dt\big(\rho_i f_i(t)\big)\cos\nu_i G1 - \frac{1}{k}\int_{t_i} dt\frac{\partial}{\partial t}\big(\rho_i f_i(t)\big)(z-z_i)G1'\right] \tag{A3}$$

$$E_\rho\left(J_{mi}^\phi\right) = \frac{-j\eta J_{mi}^\phi}{4\pi}\left[-jk\int_{t_i} dt\big(\rho_i f_i(t)\big)G3 - \frac{jm}{k\rho_i}\int_{t_i} dt\big(\rho_i f_i(t)\big)(\rho G1' - \rho_i G2')\right] \tag{A4}$$

$$E_\phi\left(J_{mi}^\phi\right) = \frac{-j\eta J_{mi}^\phi}{4\pi}\left[+k\int_{t_i} dt\big(\rho_i f_i(t)\big)G2 - \frac{m}{k\rho_i}\int_{t_i} dt\big(f_i(t)\big)\rho_i G3'\right] \tag{A5}$$

$$E_z\left(J_{mi}^\phi\right) = \frac{-j\eta J_{mi}^\phi}{4\pi}\left[-\frac{jm}{k\rho_i}\int_{t_i} dt\big(\rho_i f_i(t)\big)(z-z_i)G1'\right] \tag{A6}$$

$$E_\rho\left(M_{mi}^t\right) = \frac{-M_{mi}^t}{4\pi}\, j\int_{t_i} dt\big(\rho_i f_i(t)\big)\left[\cos\nu_i\rho_i G3' + \sin\nu_i(z-z_i)G3'\right] \tag{A7}$$

$$E_\phi\left(M_{mi}^t\right) = \frac{-M_{mi}^t}{4\pi}\int_{t_i} dt\big(\rho_i f_i(t)\big)\left[\cos\nu_i\left[\rho G1' - \rho_i G2'\right] - \sin\nu_i(z-z_i)G2'\right] \tag{A8}$$

$$E_z\left(M_{mi}^t\right) = -j\,\frac{-M_{mi}^t}{4\pi}\int_{t_i} dt(\rho_i f_i(t))\sin\nu_i\rho G_3' \tag{A9}$$

$$E_\rho\left(M_{mi}^\phi\right) = \frac{-M_{mi}^\phi}{4\pi}\int_{t_i} dt(\rho_i f_i(t))(z-z_i)G2' \tag{A10}$$

$$E_\phi\left(M_{mi}^\phi\right) = \frac{-M_{mi}^\phi}{4\pi}\int_{t_i} dt(\rho_i f_i(t))j(z-z_i)G3' \tag{A11}$$

$$E_z\left(M_{mi}^\phi\right) = \frac{-M_{mi}^\phi}{4\pi}(-1)\int_{t_i} dt\big(\rho_i f_i(t)\big)\left(\rho G2' - \rho_i G1'\right) \tag{A12}$$

where

$$\begin{aligned}
G1 &= \int_0^{2\pi}\cos m\phi' G\,d\phi'\\
G2 &= \int_0^{2\pi}\cos\phi'\cos m\phi' G\,d\phi'\\
G3 &= \int_0^{2\pi}\sin\phi'\sin m\phi' G\,d\phi'\\
G1' &= \int_0^{2\pi}\cos m\phi' G'\,d\phi'\\
G2' &= \int_0^{2\pi}\cos\phi'\cos m\phi' G'\,d\phi'\\
G3' &= \int_0^{2\pi}\sin\phi'\sin m\phi' G'\,d\phi'\\
G &= \frac{e^{-jkR}}{R}\\
G' &= (1+jkR)\,\frac{e^{-jkR}}{R^3}.
\end{aligned} \tag{A13}$$

The integrals in (A13) have no closed form and, consequently, are obtained numerically.

The electric-field expressions can now be presented as

$$E_\rho(r) = \sum_{m,i}\left[E_\rho\left(J_{mi}^t\right) + E_\rho\left(J_{mi}^\phi\right) + E_\rho\left(M_{mi}^t\right) + E_\rho\left(M_{mi}^\phi\right)\right] \tag{A14}$$

$$E_\phi(r) = \sum_{m,i}\left[E_\phi\left(J_{mi}^t\right) + E_\phi\left(J_{mi}^\phi\right) + E_\phi\left(M_{mi}^t\right) + E_\phi\left(M_{mi}^\phi\right)\right] \tag{A15}$$

$$E_z(r) = \sum_{m,i}\left[E_z\left(J_{mi}^t\right) + E_z\left(J_{mi}^\phi\right) + E_z\left(M_{mi}^t\right) + E_z\left(M_{mi}^\phi\right)\right]. \tag{A16}$$

The magnetic-field expression can be obtained using a duality theorem. From the cylindrical coordinates components, it is easy to obtain the Cartesian field components.

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